## Chapter 2 - Group Theory/Chapter 3 - Some Preliminary Lemnpaisblem 4, p. 35

Let $P(i)$ be the proposition $(a b)^{i}=a^{i} b^{i}$, for all $a, b$ in $G$, where $i$ is some integer. The problem can then be restated as $P(i) \wedge P(i+1) \wedge P(i+2) \Rightarrow$ $G$ is abelian.

To prove this, we begin by noting (see below for proof) that

$$
\begin{equation*}
P(i) \wedge P(i+1) \Rightarrow(a b)^{i}=(b a)^{i} \tag{1}
\end{equation*}
$$

and similarly

$$
\begin{equation*}
P(i+1) \wedge P(i+2) \Rightarrow(a b)^{i+1}=(b a)^{i+1} \tag{2}
\end{equation*}
$$

Now, $(a b)^{i}=(b a)^{i} \Rightarrow(a b)^{-i}=(b a)^{-i}$, applying to $(a b)^{i+1}=(b a)^{i+1}$ yields $a b=b a$.

Proof of (1):

$$
\begin{gathered}
(a b)^{i}=a^{i} b^{i} \\
(a b)^{i+1}=a^{i+1} b^{i+1}
\end{gathered}
$$

Now $(a b)^{i+1}=a(b a)^{i} b$, so we get $(b a)^{i}=a^{i} b^{i}=(a b)^{i}$.

