Let $P(i)$ be the proposition $(ab)^i = a^i b^i$, for all a, b in G, where i is some integer. The problem can then be restated as $P(i) \wedge P(i+1) \wedge P(i+2) \Rightarrow G$is abelian.

To prove this, we begin by noting (see below for proof) that

$$P(i) \wedge P(i+1) \Rightarrow (ab)^i = (ba)^i \tag{1}$$

and similarly

$$P(i+1) \wedge P(i+2) \Rightarrow (ab)^{i+1} = (ba)^{i+1} \tag{2}$$

Now, $(ab)^i = (ba)^i \Rightarrow (ab)^{-i} = (ba)^{-i}$, applying to $(ab)^{i+1} = (ba)^{i+1}$ yields $ab = ba$. $\blacksquare$

Proof of (1)**:**

$$(ab)^i = a^i b^i$$
$$(ab)^{i+1} = a^{i+1} b^{i+1}$$

Now $(ab)^{i+1} = a(ba)^i b$, so we get $(ba)^i = a^i b^i = (ab)^i$. $\blacksquare$

ISBN10: 0-471-01090-1