

Suppose $a^2 \neq e$ for all $a \neq e$, i.e. $a \neq a^{-1}$ for all $a \neq e$. Since every element in G has a unique inverse and no $a \neq e$ has e as an inverse, $P = \{\{a, a^{-1}\} | a \neq e, a \in G\}$ partitions $G - \{e\}$. Now on one hand, since P partitions $G - \{e\}$, $\|G - \{e\}\|$ is even. On the other hand, since $\|G\|$ is even, $\|G - \{e\}\|$ must be odd. We have reached a contradiction!