Chapter 2 - Group Theory/Chapter 3 - Some Preliminary Lemprablem 11, p. 35

Suppose $a^{2} \neq e$ for all $a \neq e$, i.e. $a \neq a^{-1}$ for all $a \neq e$. Since every element in $G$ has a unique inverse and no $a \neq e$ has $e$ as an inverse, $P=$ $\left\{\left\{a, a^{-1}\right\} \mid a \neq e, a \in G\right\}$ partitions $G-\{e\}$. Now on one hand, since $P$ partitions $G-\{e\},\|G-\{e\}\|$ is even. On the other hand, since $\|G\|$ is even, $\|G-\{e\}\|$ must be odd. We have reached a contradiction!

