Exercise 4 (*page 85*)

Since $\dim(\Lambda^n(\mathbb{R})) = 1$, and since $\omega \in \Lambda^n(V)$ implies that $f^*\omega \in \Lambda^n(\mathbb{R}^n)$, we know that $f^*\omega = c\det$ for some $c \in \mathbb{R}$. To find out c, we evaluate this equation at $(e_1, \ldots, e_n)$:

$$f^*\omega(e_1, \ldots, e_n) = c\det\begin{pmatrix} e_1 \\ \vdots \\ e_n \end{pmatrix} = c$$

or

$$\omega(f(e_1), \ldots, f(e_n)) = c$$

Applying Theorem 4-6, we get

$$det(a_{ij}) \cdot \omega(v_1, \ldots, v_n) = c$$

where $\{v_i\}_{i=1}^n$ is a basis for v and the a_{ij} are defined by $f(e_i) = \sum_{j=1}^n a_{ij}v_j$. By hypothesis, $[f(e_1), \ldots, f(e_n)] = \mu$ so that $\det(a_{ij}) = 1$. Also, since ω is the volume element w.r.t. μ and T, we get $\omega(v_1, \ldots, v_n) = 1$, so we conclude $c = 1$. Thus $f^*\omega = \det$ follows.

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