## Chapter 4 - Integration on Chains/Chapter 1 - Algebraic Prelimpnablemn 4, p. 85

Since $\operatorname{dim}\left(\Lambda^{n}(\mathbb{R})\right)=1$, and since $\omega \in \Lambda^{n}(V)$ implies that $f^{*} \omega \in \Lambda^{n}\left(\mathbb{R}^{n}\right)$, we know that $f^{*} \omega=c$ det for some $c \in \mathbb{R}$. To find out $c$, we evaluate this equation at $\left(e_{1}, \ldots, e_{n}\right)$ :

$$
f^{*} \omega\left(e_{1}, \ldots, e_{n}\right)=c \operatorname{det}\left(\begin{array}{c}
e_{1} \\
\vdots \\
e_{n}
\end{array}\right)=c
$$

or

$$
\omega\left(f\left(e_{1}\right), \ldots, f\left(e_{n}\right)\right)=c
$$

Applying Theorem 4-6, we get

$$
\operatorname{det}\left(a_{i j}\right) \cdot \omega\left(v_{1}, \ldots, v_{n}\right)=c
$$

where $\left\{v_{i}\right\}_{i=1}^{n}$ is a basis for $v$ and the $a_{i j}$ are defined by $f\left(e_{i}\right)=\sum_{j=1}^{n} a_{i j} v_{j}$. By hypothesis, $\left[f\left(e_{1}\right), \ldots, f\left(e_{n}\right)\right]=\mu$ so that $\operatorname{det}\left(a_{i j}\right)=1$. Also, since $\omega$ is the volume element w.r.t. $\mu$ and $T$, we get $\omega\left(v_{1}, \ldots, v_{n}\right)=1$, so we conclude $c=1$. Thus $f^{*} \omega=\operatorname{det}$ follows.

