

Since  $\dim(\Lambda^n(\mathbb{R})) = 1$ , and since  $\omega \in \Lambda^n(V)$  implies that  $f^*\omega \in \Lambda^n(\mathbb{R}^n)$ , we know that  $f^*\omega = c \det$  for some  $c \in \mathbb{R}$ . To find out  $c$ , we evaluate this equation at  $(e_1, \dots, e_n)$ :

$$f^*\omega(e_1, \dots, e_n) = c \det \begin{pmatrix} e_1 \\ \vdots \\ e_n \end{pmatrix} = c$$

or

$$\omega(f(e_1), \dots, f(e_n)) = c$$

Applying Theorem 4-6, we get

$$\det(a_{ij}) \cdot \omega(v_1, \dots, v_n) = c$$

where  $\{v_i\}_{i=1}^n$  is a basis for  $v$  and the  $a_{ij}$  are defined by  $f(e_i) = \sum_{j=1}^n a_{ij}v_j$ . By hypothesis,  $[f(e_1), \dots, f(e_n)] = \mu$  so that  $\det(a_{ij}) = 1$ . Also, since  $\omega$  is the volume element w.r.t.  $\mu$  and  $T$ , we get  $\omega(v_1, \dots, v_n) = 1$ , so we conclude  $c = 1$ . Thus  $f^*\omega = \det$  follows.