To show *existence*, define $\lambda = \int \omega = \int_0^1 f dx$, and $g(x) = \int_0^x f dx - \lambda x$. By Theorem 4-7, we calculate $dg = Dg \cdot dx$. $Dg = f - \lambda$, so $dg = \omega - \lambda dx$ follows. Also note that $g(0) = \int_0^0 f dx - \lambda \cdot 0 = 0$, and $g(1) = \int_0^1 f dx - \lambda = \int_0^1 f dx - \int_0^1 f dx = 0$, so $g(0) = g(1) = 0$.

To show *uniqueness*, suppose that $\omega - \lambda dx = dg$, where $g(0) = g(1)$. Integrating, we get

$$\int_0^1 \omega - \lambda = \int_0^1 dg$$

By Stoke's theorem, we have $\int_0^1 dg = g(1) - g(0) = 0$, so $\lambda = \int_0^1 \omega = \int_0^1 f dx$. So the λ is completely determined by f, and is thus *unique*.

 ISBN10: 0-8053-9021-9