Exercise 3 (*page 12*)

Yes, it is true. Suppose $x+y$, $y+z$ and $x+z$ were not linearly independent. For instance, suppose $\exists \alpha, \beta$ s.t. $\alpha(x+y)+\beta(y+z) = x+z$. Then we'd have

$$\alpha(x+y)+\beta(y+z) = x+z$$

$$(1-\alpha)x+(1-\beta)z = y(\alpha+\beta)$$

If any two of $1-\alpha$, $1-\beta$ and $\alpha+\beta$ are zero, then the third one is not zero:

$$1-\alpha = 0 \text{ and } 1-\beta = 0 \Rightarrow \alpha+\beta = 2$$

$$1-\alpha = 0 \text{ and } \alpha+\beta = 0 \Rightarrow 1-\beta = 2$$

$$1-\beta = 0 \text{ and } \alpha+\beta = 0 \Rightarrow 1-\alpha = 2$$

Therefore $\{x, y, z\}$ are linearly dependent. ∎

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