Suppose $a \in G$ is an element s.t. $a^2 \neq a$. Then $\exists n \geq 1$ s.t. $a^{n+1} = a$, since G is finite. We claim that a^n satisfies $a^n b = b a^n = b$, for all $b \in G$. This is true since

$$a^{n+1} = a$$
$$a^{n+1} b = ab$$
$$a^n b = b$$

Where the last equality follows from left-cancellation.

Now $ba^n = b$ follows similarly. Therefore a^n is the identity element.

It is easy to see that a^{n-1} is both the left and right inverse of a.

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