## Exercise 6 (*page 19*)

By theorem 2, we can find a basisi of $\mathfrak{V}$ s.t. $\{x_1, \cdots, x_m, y_1, \cdots, y_{N-m}\}$, where $N = \dim(\mathfrak{V})$, $\{x_1, \cdots, x_m\}$ is a basis for $\mathfrak{m}$ and $m \neq 0, N$. It is clear that $\mathfrak{n} = \text{span} y_i$ is a complement to $\mathfrak{m}$. Now let $y = a_1 x_1 + \cdots + a_m x_m + b_1 y_1 + \cdots + b_{N-m} x_{N-m}$ where not all $a_i$ and not all $b_i$ are zero. This means that $y \notin \mathfrak{m}$ and $y \notin \mathfrak{n}$. Suppose, without loss of generality, that $b_1 \neq 0$. Then $\{x_1, \cdots, x_m, y, y_2, \cdots, y_{N-m}\}$ also spans $\mathfrak{V}$, and so $\{y, y_2, \cdots, y_{N-m}\}$ is a different complement to $\mathfrak{m}$. Thus, the answer to **(a)** is “no”. As for **(b)**, the only thing to note is that if $\{x_1, \cdots, x_m\}$ is a basis for $\mathfrak{m}$ and $\{y_1, \cdots, y_n\}$ is a basis for $\mathfrak{n}$, and if $\mathfrak{m}$ and $\mathfrak{n}$ are complements, then clearly $\{x_1, \cdots, x_m, y_1, \cdots, y_n\}$ spans $\mathfrak{V}$, and furthermore this set is linearly independent since the $\{x_i\}$ and $\{y_i\}$ are, and since $\mathfrak{m} \cap \mathfrak{n} = \mathfrak{o}$. Thus $\dim \mathfrak{V} = m + n$. ∎

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