Exercise 7 (*page 4*)

(a) Suppose T preserves inner product. Then $|Tx| = \sqrt{\langle Tx, Tx\rangle} = \sqrt{\langle x, x\rangle} = |x|$. Thus T is norm preserving. Now suppose T preserves norm. Then by theorem 1-2(5), we have $\langle Tx, Ty\rangle = \frac{|Tx+Ty|^2 - |Tx-Ty|^2}{4} = \frac{|T(x+y)|^2 - |T(x-y)|^2}{4} = \frac{|x+y|^2 - |x-y|^2}{4} = \langle x, y\rangle$, and so T preserves inner product. **(b)** If T was not 1-to-1, then there would exist x, y with $x \neq y$ (i.e. $|x-y| > 0$), yet with $Tx = Ty$ (i.e. $|Tx - Ty| = 0$. But by hypothesis $|Tx - Ty| = |x-y| = 0$, so T must have an inverse, which is clearly also norm preserving (and therefore it also preserves the inner product).

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