Let $U \subset Y$ be an open set, and let $p \in f^{-1}(U)$. There is an m such that $p \in A_m$. Note that $(f|_{A_{m+1}})^{-1}(U) = f^{-1}(U) \cap A_{m+1}$, which is open in A_{m+1}. This means that there is an open set $V \subset X$ such that $V \cap A_{m+1} = f^{-1}(U) \cap A_{m+1}$. Define $W = V \cap A^{\circ}_{m+1}$, which is open in X. We know that $p \in A_m \subset A^{\circ}_{m+1}$, and $p \in f^{-1}(U) \cap A_{m+1} = V \cap A_{m+1}$ implies that $p \in V$, so we can conclude $p \in W$. Finally, note that $W = V \cap A^{\circ}_{m+1} \subset V \cap A_{m+1} = f^{-1}(U) \cap A_{m+1} \subset f^{-1}(U)$, so we conclude $W \subset f^{-1}(U)$. Thus, W is an open set containing p, and is contained in $f^{-1}(U)$. We conclude that $f^{-1}(U)$ is open, for any open $U \subset Y$, and therefore f is continuous.

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