

Let  $I$  be the (compact) interval, and let  $S \subset I$  be infinite. Suppose  $S$  has no limit points in  $I$ . Then there is an open  $U_x$  for each  $x \in I$  such that  $x \in U_x$  and  $(U_x - \{x\}) \cap S = \emptyset$ . Since  $\{U_x | x \in I\}$  forms an open cover of  $I$ , Heine-Borel tells us that there are  $x_1, \dots, x_n$  such that  $I \subset U_{x_1} \cup \dots \cup U_{x_n}$ . Each  $U_{x_i}$  contains at most one point from  $S$  and therefore  $S$  must be finite – a contradiction.