Let $\mathfrak{B}=\{(-\infty, a): a \in \mathbb{R}\} \cup\{\emptyset\} \cup\{\mathbb{R}\}$. By theorem 2.5, this forms the basis for some topology on $\mathbb{R}$. In this topology, $[0,1]$ is compact, but it's closure is $[0, \infty)$, which is not compact.

