

(a) Define $g : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ as $g(x, y) = (f(x, y), y)$. The differential is then

$$g'(x, y) = \begin{pmatrix} D_1f(x, y) & D_2f(x, y) \\ 0 & 1 \end{pmatrix}$$

and so $\det g'(x, y) = D_1f(x, y)$. Now, if f is 1 – 1, then there must exist some point (a, b) at which $\det g'(a, b) \neq 0$.